## Georgia <br> Standards of Excellence Course Curriculum Overview <br> Mathematics

## GSE Grade 7



Richard Woods, Georgia's School Superintendent "Educating Georgia's Future"

## Georgia Department of Education

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## GSE Mathematics | Grade 7 | Curriculum Map

| GSE Grade 7 Curriculum Map |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}^{\text {st }}$ Semester $\quad$ l |  |  |  |  | $2^{\text {nd }}$ Semester |  |
| Unit 1 <br> (4-5 weeks) | $\begin{gathered} \text { Unit } 2 \\ (4-5 \text { weeks }) \end{gathered}$ | $\begin{gathered} \text { Unit } 3 \\ (4-5 \text { weeks }) \end{gathered}$ | Unit 4 $(4-5$ weeks) | $\begin{gathered} \text { Unit } 5 \\ (4-5 \text { weeks }) \end{gathered}$ | Unit 6 (3-4 weeks) | Unit 7 <br> (3-4 weeks) |
| Operations with Rational Numbers | Expressions and Equations | Ratios and Proportional | Geometry | Inferences | Probability | Show What We Know |
| MGSE7.NS.1a MGSE7.NS.1b MGSE7.NS.1c MGSE7.NS.1d MGSE7.NS.2a MGSE7.NS.2b MGSE7.NS.2c MGSE7.NS.2d MGSE7.NS. 3 | MGSE7.EE. 1 MGSE7.EE. 2 MGSE7.EE. 3 MGSE7.EE.4a MGSE7.EE.4b MGSE7.EE.4c | MGSEE7.RP. 1 MGSE7.RP.2a MGSE7.RP.2b MGSE7.RP.2c MGSE7.RP.2d MGSE7.RP. 3 MGSE7.G. 1 | MGSE7.G. 2 MGSE7.G. 3 MGSE7.G.4 MGSE7.G.5 MGSE7.G. 6 | MGSE7.SP. 1 MGSE7.SP. 2 MGSE7.SP. 3 MGSE7.SP. 4 | MGSE7.SP. 5 <br> MGSE7.SP. 6 <br> MGSE7.SP7 <br> MGSE7.SP.7a <br> MGSE7.SP.7b <br> MGSE7.SP.8a <br> MGSE7.SP.8b <br> MGSE7.SP.8c | ALL |
| These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units. All units will include the Mathematical Practices and indicate skills to maintain. |  |  |  |  |  |  |

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.

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Grades 6-8 Key:
NS = The Number System
RP = Ratios and Proportional Relationships
EE = Expressions and Equations
G = Geometry
SP = Statistics and Probability.
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$7^{\text {th }}$ Grade Course Curriculum Overview

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The Comprehensive Course Overviews are designed to provide access to multiple sources of support for implementing and instructing courses involving the Georgia Standards of Excellence.

## GSE Mathematics | Grade 7 Critical Areas

The middle school standards specify the mathematics that all students should study in order to be high school ready. The middle school standards are listed in conceptual categories including Number Sense, Algebra, Expressions and Equations, Geometry, and Statistics and Probability.

In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two and three-dimensional shapes to solve problems involving area, surface area, and volume, and (4) drawing inferences about populations based on samples.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight standards for mathematical practice should be addressed constantly as well. To assure that these units are taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under "Evidence of Learning" be reviewed early in the planning process. A variety of resources should be utilized to supplement these units. These units provide much needed content information, but excellent learning activities as well. The tasks in theseunits illustrate the types of learning activities that should be utilized from a variety of sources.

## GSE Mathematics Grade 7 Unit Descriptions

The fundamental purpose of seventh grade mathematics is to formalize and extend the mathematics that students learned in the previous grades. The critical areas, organized into units, deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Seventh grade standards use algebra to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Standards for Mathematical Practice apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Unit 1 This unit builds upon the students understanding of rational numbers that was developed in $6^{\text {th }}$ grade. In Grade 7, learning now moves to exploring and ultimately formalizing rules for operations (addition, subtraction, multiplication and division) with integers. Using both contextual and numerical problems, students should explore what happens when negative numbers and positive numbers are combined. Repeated opportunities over time will allow students to compare the results of adding, subtracting, multiplying and dividing pairs of numbers, leading to the generalization of the rules. Fractional rational numbers and whole

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numbers should be used in computations and explorations. Students will be able to give contextual examples of integer operations, write and solve equations for real-world problems and explain how the properties of operations apply. Real-world situations could include: profit/loss, money, weight, sea level, debit/credit, football yardage, etc.

Unit 2 This unit builds on what that the students learned in $6^{\text {th }}$ grade regarding mathematical properties. They will continue to build on their knowledge of order of operations and other mathematical properties, and use these properties of operations to rewrite equivalent numerical expressions. The students should continue to use properties that were used with whole numbers in Grade 6 and understand that these properties apply to integers, rational and real numbers as well. Students will also have the opportunity to write expressions and equations in more than one format and understand that they are still equal. They will be given the opportunity to use variables to represent quantities in real-world problems.

Unit 3 This unit builds on the students' knowledge and understandings of rate and unit concepts that were developed in Grade 6. This includes the need to develop proportional relationships through the analysis of graphs, tables, equations, and diagrams. Grade 7 will push for the students' to develop a deep understanding of the characteristics of a proportional relationship. Mathematics should be represented in as many ways as possible in this unit by using graphs, tables, pictures, symbols and words. Some examples of providing the students with this opportunity are the following: researching newspaper ads, constructing their own questions, keeping a log of prices (particularly sales) and determining savings by purchasing items on sale.

Unit 4 This unit focuses on how to teach students to draw geometric figures using rulers and protractor with an emphasis on triangles. Students will also explore two-dimensional crosssections of cylinders, cones, pyramids, and prisms. Their knowledge from $6^{\text {th }}$ grade will help when they are learning to write and solve equations involving angle relationships and when solving engaging problems that require determining the area, volume, and surface area of fundamental solid figures. This unit also requires students to know and use the formula for the circumference and area of a circle.

Unit 5 This unit builds on students' knowledge and understanding of statistics from the $6^{\text {th }}$ grade. Students begin to use random samples to make predictions about an entire population and judge the possible discrepancies of the predictions. Opportunities are provided for students to use real-life situations from science and social studies to show the purpose for using random sampling to make inferences about a population.

Unit 6 In this unit, students will begin to understand the probability of chance (simple and compound). Along with the understanding of probability, they will develop probability models to be used to find the probability of events. They will make predictions and use the information from simulations for predictions. The students will begin to expand their knowledge and understanding of the probability of simple events.

## Flipbooks

These "FlipBooks" were developed by the Kansas Association of Teachers of Mathematics (KATM) and are a compilation of research, "unpacked" standards from many states, instructional strategies and examples for each standard at each grade level. The intent is to show the connections to the Standards of Mathematical Practices for the content standards and to get detailed information at each level. The Grade 7 Flipbook is an interactive document arranged by the content domains listed on the following pages. The links on each domain and standard will take you to specific information on that standard/domain within the Flipbook.

## Analyze proportional relationships and use them to solve real-world and mathematical problems.

MGSE7.RP. 1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $1 / 2$ mile in each $1 / 4$ hour, compute the unit rate as the complex fraction $(1 / 2) /(1 / 4)$ miles per hour, equivalently 2 miles per hour.

MGSE7.RP. 2 Recognize and represent proportional relationships between quantities.
MGSE7.RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

MGSE7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

MGSE7.RP.2c Represent proportional relationships by equations. For example, if total cost $t$ is proportional to the number $n$ of items purchased at a constant price $p$, the relationship between the total cost and the number of items can be expressed as $t=p n$.

MGSE7.RP.2d Explain what a point $(x, y)$ on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1, r)$ where r is the unit rate.

MGSE7.RP. 3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, and fees.

## Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

MGSE7.NS. 1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

MGSE7.NS.1a Show that a number and its opposite have a sum of 0 (are additive inverses). Describe situations in which opposite quantities combine to make 0. For example, your bank account balance is -\$25.00. You deposit $\$ 25.00$ into your account. The net balance is $\$ 0.00$.

MGSE7.NS.1b Understand $p+q$ as the number located a distance $|q|$ from $p$, in the positive or negative direction depending on whether $q$ is positive or negative. Interpret sums of rational numbers by describing real world contexts.
MGSE7.NS.1c Understand subtraction of rational numbers as adding the additive inverse, $p-q=p+(-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
MGSE7.NS.1d Apply properties of operations as strategies to add and subtract rational numbers.

MGSE7.NS. 2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

MGSE7.NS.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1)=1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

MGSE7.NS.2b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If $p$ and $q$ are integers then $-(p / q)=(-p) / q=p /(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
MGSE7.NS.2c Apply properties of operations as strategies to multiply and divide rational numbers.

MGSE7.NS.2d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0 s or eventually repeats.

MGSE7.NS. 3 Solve real-world and mathematical problems involving the four operations with rational numbers.

## Use properties of operations to generate equivalent expressions.

MGSE7.EE. 1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

MGSE7.EE. 2 Understand that rewriting an expression in different forms in a problem context can clarify the problem and how the quantities in it are related. For example $a+0.05 a=1.05 a$ means that adding a 5\% tax to a total is the same as multiplying the total by 1.05.

## Solve real-life and mathematical problems using numerical and algebraic expressions and equations

MGSE7.EE. 3 Solve multistep real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals) by applying properties of operations as strategies to calculate with numbers, converting between forms as appropriate, and assessing the reasonableness of answers using mental computation and estimation strategies.
For example:

- If a woman making $\$ 25$ an hour gets a $10 \%$ raise, she will make an additional $1 / 10$ of her salary an hour, or $\$ 2.50$, for a new salary of $\$ 27.50$.
- If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

MGSE7.EE. 4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

MGSE7.EE.4a Solve word problems leading to equations of the form $p x+q=r$ and $p(x$ $+q)=r$, where $p, q$, and $r$ are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm . What is its width?

MGSE7.EE.4b Solve word problems leading to inequalities of the form $p x+q>r$ or $p x+$ $q<r$, where $p, q$, and $r$ are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example, as a salesperson, you are paid $\$ 50$ per week plus $\$ 3$ per sale. This week you want your pay to be at least $\$ 100$. Write an inequality for the number of sales you need to make, and describe the solutions.

MGSE7.EE.4c Solve real-world and mathematical problems by writing and solving equations of the form $\mathrm{x}+\mathrm{p}=\mathrm{q}$ and $\mathrm{px}=\mathrm{q}$ in which p and q are rational numbers.

## Geometry

Draw, construct, and describe geometrical figures and describe the relationships between them.

MGSE7.G. 1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

MGSE7.G. 2 Explore various geometric shapes with given conditions. Focus on creating triangles from three measures of angles and/or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

MGSE7.G. 3 Describe the two-dimensional figures (cross sections) that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms, right rectangular pyramids, cones, cylinders, and spheres.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

MGSE7.G. 4 Given the formulas for the area and circumference of a circle, use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

MGSE7.G. 5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

MGSE7.G. 6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

## Use random sampling to draw inferences about a population.

MGSE7.SP. 1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

MGSE7.SP. 2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

## Draw informal comparative inferences about two populations.

MGSE7.SP. 3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the medians by expressing it as a multiple of the interquartile range.

MGSE7.SP. 4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

## Investigate chance processes and develop, use, and evaluate probability models.

MGSE7.SP. 5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $1 / 2$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
MGSE7.SP. 6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency. Predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
MGSE7.SP. 7 Develop a probability model and use it to find probabilities of events. Compare experimental and theoretical probabilities of events. If the probabilities are not close, explain possible sources of the discrepancy.

MGSE7.SP.7a Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
MGSE7.SP.7b Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

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MGSE7.SP. 8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

MGSE7.SP.8a Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
MGSE7.SP.8b Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

MGSE7.SP.8c Explain ways to set up a simulation and use the simulation to generate frequencies for compound events. For example, if $40 \%$ of donors have type A blood, create a simulation to predict the probability that it will take at least 4 donors to find one with type A blood?

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## Mathematics | Standards for Mathematical Practice

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction. The BLUE links will provide access to classroom videos on each standard for mathematical practice accessed on the Inside Math website.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

## 1. Make sense of problems and persevere in solving them.

In grade 7, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"

## 2. Reason abstractly and quantitatively.

In grade 7, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

## 3. Construct viable arguments and critique the reasoning of others.

In grade 7, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.

## 4. Model with mathematics.

In grade 7, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students explore covariance and represent two quantities simultaneously. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences, make comparisons and formulate

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predictions. Students use experiments or simulations to generate data sets and create probability models. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.

## 5. Use appropriate tools strategically.

Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Students might use physical objects or applets to generate probability data and use graphing calculators or spreadsheets to manage and represent data in different forms.

## 6. Attend to precision.

In grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, specify units of measure, and label axes accurately. Students use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations or inequalities.

## 7. Look for and make use of structure.

Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables making connections between the constant of proportionality in a table with the slope of a graph. Students apply properties to generate equivalent expressions (i.e. $6+2 x=3(2+x)$ by distributive property) and solve equations (i.e. $2 c+3=15,2 c=12$ by subtraction property of equality), $\mathrm{c}=6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving scale drawings, surface area, and volume. Students examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities.

## 8. Look for and express regularity in repeated reasoning.

In grade 7, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a / b \div c / d=a d / b c$ and construct other examples and models that confirm their generalization. They extend their thinking to include complex fractions and rational numbers. Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities. They create, explain, evaluate, and modify probability models to describe simple and compound events.

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## Connecting the Standards for Mathematical Practice to the Content Standards

The Standards for Mathematical Practice describe ways in which developing student practioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

## Classroom Routines

The importance of continuing the established classroom routines cannot be overstated. Daily routines must include such obvious activities as estimating, analyzing data, describing patterns, and answering daily questions. They should also include less obvious routines, such as how to select materials, how to use materials in a productive manner, how to put materials away, how to access classroom technology such as computers and calculators. An additional routine is to allow plenty of time for children to explore new materials before attempting any directed activity with these new materials. The regular use of routines is important to the development of students' number sense, flexibility, fluency, collaborative skills and communication. These routines contribute to a rich, hands-on standards based classroom and will support students' performances on the tasks in this unit and throughout the school year.

## Strategies for Teaching and Learning

- Students should be actively engaged by developing their own understanding.
- Mathematics should be represented in as many ways as possible by using graphs, tables, pictures, symbols and words.
- Interdisciplinary and cross curricular strategies should be used to reinforce and extend the learning activities.
- Appropriate manipulatives and technology should be used to enhance student learning.


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- Students should be given opportunities to revise their work based on teacher feedback, peer feedback, and metacognition which includes self-assessment and reflection.
- Students should write about the mathematical ideas and concepts they are learning.
- Consideration of all students should be made during the planning and instruction of this unit. Teachers need to consider the following:
-What level of support do my struggling students need in order to be
successful with this nit?
-In what way can I deepen the understanding of those students who are competent in this unit?
-What real life connections can I make that will help my students utilize the skills practiced in this unit?


## Tasks

The following tasks represent the level of depth, rigor, and complexity expected of all seventh grade students. These tasks, or tasks of similar depth and rigor, should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they may also be used for teaching and learning (learning/scaffolding task).

| Scaffolding Task | Tasks that build up to the learning task. |
| :--- | :--- |
| Learning Task | Constructing understanding through deep/rich contextualized problem <br> solving tasks. |
| Practice Task | Tasks that provide students opportunities to practice skills and concepts. |
| Performance Task | Tasks which may be a formative or summative assessment that checks for <br> student understanding/misunderstanding and or progress toward <br> the standard/learning goals at different points during a unit of <br> instruction. |
| Culminating Task | Designed to require students to use several concepts learned during the <br> unit to answer a new or unique situation. Allows students to give evidence <br> of their own understanding toward the mastery of the standard and <br> requires them to extend their chain of mathematical reasoning. |
| Short Cycle Task | Designed to exemplify the performance targets that the standards imply. <br> The tasks, with the associated guidance, equip teachers to monitor overall <br> progress in their students’ mathematics. |
| Formative <br> Assessment Lesson <br> (FAL) | Lessons that support teachers in formative assessment which both reveal <br> and develop students' understanding of key mathematical ideas and <br> applications. These lessons enable teachers and students to monitor in more <br> detail their progress towards the targets of the standards. |
| information on |  |$\quad$| A Three-Act Task is a whole group mathematics task consisting of 3 |
| :--- |
| distinct parts: an engaging and perplexing Act One, an information and |
| solution seeking Act Two, and a solution discussion and solution revealing |
| Act Three. |

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Achieve CCSS- CTE Classroom Tasks

Designed to demonstrate how the Common Core and Career and Technical Education knowledge and skills can be integrated. The tasks provide teachers with realistic applications that combine mathematics and CTE content.

## Formative Assessments Lessons (FALs)

What is a Formative Assessment Lesson (FAL)? The Formative Assessment Lesson is designed to be part of an instructional unit typically implemented approximately two-thirds of the way through the instructional unit. The results of the tasks should then be used to inform the instruction that will take place for the remainder of the unit.

Formative Assessment Lessons are intended to support teachers in formative assessment. They both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

What does a Formative Assessment Lesson look like in action? Videos of Georgia Teachers implementing FALs can be accessed HERE and a sample of a FAL lesson may be seen HERE

Where can I find more information on FALs? More information on types of Formative Assessment Lessons, their use, and their implementation may be found on the Math Assessment Project's guide for teachers.

## Where can I find samples of FALs?

## Formative Assessment Lessons can also be found at the following sites:

Mathematics Assessment Project
Kenton County Math Design Collaborative
MARS Tasks by grade level
A sample FAL with extensive dialog and suggestions for teachers may be found HERE. This resource will help teachers understand the flow and purpose of a FAL.

Where can I find more training on the use of FALs? The Math Assessment Project has developed Professional Development Modules that are designed to help teachers with the practical and pedagogical challenges presented by these lessons.

Module 1 introduces the model of formative assessment used in the lessons, its theoretical background and practical implementation. Modules 2 \& $\underline{3}$ look at the two types of Classroom Challenges in detail. Modules $4 \& \underline{5}$ explore two crucial pedagogical features of the lessons: asking probing questions and collaborative learning.

Georgia RESA's may be contacted about professional development on the use of FALs in the classroom. The request should be made through the teacher's local RESA and can be referenced by asking for more information on the Mathematics Design Collaborative (MDC).

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## Spotlight Tasks

A Spotlight Task has been added to each MGSE mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Georgia Standards of Excellence, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and ProblemBased Learning from Robert Kaplinsky.

## 3-Act Tasks

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

## Guidelines for 3-Act Tasks and Patient Problem Solving (Teaching without the Textbook) Adapted from Dan Meyer

## Developing the mathematical Big Idea behind the 3-Act task:

- Create or find/use a clear visual which tells a brief, perplexing mathematical story. Video or live action works best. (See resource suggestions in the Guide to 3-Act Tasks)
- Video/visual should be real life and allow students to see the situation unfolding.
- Remove the initial literacy/mathematics concerns. Make as few language and/or math demands on students as possible. You are posing a mathematical question without words.
- The visual/video should inspire curiosity or perplexity which will be resolved via the mathematical big idea(s) used by students to answer their questions. You are creating an intellectual need or cognitive dissonance in students.


## Enacting the 3-Act in the Classroom

## Act 1 (The Question):

Set up student curiosity by sharing a scenario:

- Teacher says, "I'm going show you something I came across and found interesting" or,
"Watch this."
- Show video/visual.
- Teacher asks, "What do you notice/wonder?" and "What are the first questions that come to mind?"
- Students share observations/questions with a partner first, then with the class (Think-Pair-Share).
- Students have ownership of the questions because they posed them.
- Leave no student out of this questioning. Every student should have access to the scenario. No language or mathematical barriers. Low barrier to entry.
-Teacher records questions (on chart paper or digitally-visible to class) and ranks them by popularity.


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- Determine which question(s) will be immediately pursued by the class. If you have a particular question in mind, and it isn't posed by students, you may have to do some skillful prompting to orient their question to serve the mathematical end. However, a good video should naturally lead to the question you hope they'll ask. You may wish to pilot your video on colleagues before showing it to students. If they don't ask the question you are after, your video may need some work.
- Teacher asks for estimated answers in response to the question(s). Ask first for best estimates, then request estimates which are too high and too low. Students are no defining and defending parameters for making sense of forthcoming answers.
- Teacher asks students to record their actual estimation for future reference.


## Act 2 (Information Gathering):

Students gather information, draw on mathematical knowledge, understanding, and resources to answer the big question(s) from Act-1:

- Teacher asks, "What information do you need to answer our main question?"
- Students think of the important information they will need to answer their questions.
- Ask, "What mathematical tools do you have already at your disposal which would be useful in answering this question?"
- What mathematical tools might be useful which students don't already have? Help them develop those.
- Teacher offers smaller examples and asks probing questions.
o What are you doing?
o Why are you doing that?
o What would happen if...?
o Are you sure? How do you know?


## Act 3 (The Reveal):

The payoff.

- Teacher shows the answer and validates students' solutions/answer.
- Teacher revisits estimates and determines closest estimate.
- Teacher compares techniques, and allows students to determine which is most efficient.


## The Sequel:

- Students/teacher generalize the math to any case, and "algebrafy" the problem.
- Teacher poses an extension problem- best chance of student engagement if this extension connects to one of the many questions posed by students which were not the focus of Act 2, or is related to class discussion generated during Act 2.
- Teacher revisits or reintroduces student questions that were not addressed in Act 2.


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## Why Use 3-Act Tasks? A Teacher's Response

The short answer: It's what's best for kids! If you want more, read on:

The need for students to make sense of problems can be addressed through tasks like these. The challenge for teachers is, to quote Dan Meyer, "be less helpful." (To clarify, being less helpful means to first allow students to generate questions they have about the picture or video they see in the first act, then give them information as they ask for it in act 2.) Less helpful does not mean give these tasks to students blindly, without support of any kind!

This entire process will likely cause some anxiety (for all). When jumping into 3-Act tasks for the first (second, third, .. .) time, students may not generate the suggested question. As a matter of fact, in this task about proportions and scale, students may ask many questions that are curious questions, but have nothing to do with the mathematics you want them to investigate. One question might be "How is that ball moving by itself?" It's important to record these and all other questions generated by students. This validates students' ideas. Over time, students will become accustomed to the routine of 3-act tasks and come to appreciate that there are certain kinds of mathematically answerable questions - most often related to quantity or measurement.

These kinds of tasks take time, practice and patience. When presented with options to use problems like this with students, the easy thing for teachers to do is to set them aside for any number of "reasons." I've highlighted a few common "reasons" below with my commentary (in blue):

- This will take too long. I have a lot of content to cover. (Teaching students to think and reason is embedded in mathematical content at all levels - how can you not take this time)
- They need to be taught the skills first, then maybe I'll try it. (An important part of learning mathematics lies in productive struggle and learning to persevere [SMP 1]. What better way to discern what students know and are able to do than with a mathematical context [problem] that lets them show you, based on the knowledge they already have - prior to any new information. To quote John Van de Walle, "Believe in kids and they will, flat out, amaze you!")
- My students can't do this. (Remember, whether you think they can or they can't, you're right!) (Also, this expectation of students persevering and solving problems is in every state's standards - and was there even before common core!)
- I'm giving up some control. (Yes, and this is a bit scary. You're empowering students to think and take charge of their learning. So, what can you do to make this less scary? Do what we expect students to do:
o Persevere. Keep trying these and other open-beginning, -middle, and -ended problems. Take note of what's working and focus on it!
o Talk with a colleague (work with a partner). Find that critical friend at school, another school, online.. .
o Question (use \#MTBoS on Twitter, or blogs, or Google: 3-act tasks).


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The benefits of students learning to question, persevere, problem solve, and reason mathematically far outweigh any of the reasons (read excuses) above. The time spent up front, teaching through tasks such as these and other open problems, creates a huge pay-off later on. However, it is important to note, that the problems themselves are worth nothing without teachers setting the expectation that students: question, persevere, problem solve, and reason mathematically on a daily basis. Expecting these from students, and facilitating the training of how to do this consistently and with fidelity is principal to success for both students and teachers.

Yes, all of this takes time. For most of my classes, mid to late September (we start school at the beginning of August) is when students start to become comfortable with what problem solving really is. It's not word problems - mostly. It's not the problem set you do after the skill practice in the textbook. Problem solving is what you do when you don't know what to do! This is difficult to teach kids and it does take time. But it is worth it! More on this in a future blog!

## Tips:

One strategy I've found that really helps students generate questions is to allow them to talk to their peers about what they notice and wonder first (Act 1). Students of all ages will be more likely to share once they have shared and tested their ideas with their peers. This does take time. As you do more of these types of problems, students will become familiar with the format and their comfort level may allow you to cut the amount of peer sharing time down before group sharing.

What do you do if they don't generate the question suggested? Well, there are several ways that this can be handled. If students generate a similar question, use it. Allowing students to struggle through their question and ask for information is one of the big ideas here. Sometimes, students realize that they may need to solve a different problem before they can actually find what they want. If students are way off, in their questions, teachers can direct students, carefully, by saying something like:"You all have generated some interesting questions. I'm not sure how many we can answer in this class. Do you think there's a question we could find that would allow us to use our knowledge of mathematics to find the answer to (insert quantity or measurement)?" Or, if they are really struggling, you can, again carefully, say "You know, I gave this problem to a class last year (or class, period, etc.) and they asked (insert something similar to the suggested question here). What do you think about that?" Be sure to allow students to share their thoughts.

After solving the main question, if there are other questions that have been generated by students, it's important to allow students to investigate these as well. Investigating these additional questions validates students' ideas and questions and builds a trusting, collaborative learning relationship between students and the teacher.

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Overall, we're trying to help our students mathematize their world. We're best able to do that when we use situations that are relevant (no dog bandanas, please), engaging (create an intellectual need to know), and perplexing. If we continue to use textbook type problems that are too helpful, uninteresting, and let's face it, perplexing in all the wrong ways, we're not doing what's best for kids; we're training them to not be curious, not think, and worst of all . . . dislike math.

## 3-Act Task Resources:

- www.estimation180.com
- www.visualpatterns.org
- 101 Questions
- Dan Meyer's 3-Act Tasks
- 3-Act Tasks for Elementary and Middle School
- Andrew Stadel
- Jenise Sexton
- Graham Fletcher
- Fawn Nguyen
- Robert Kaplinsky
- Open Middle
- Check out the Math Twitter Blog-o-Sphere (MTBoS) - you'll find tons of support and ideas!


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## Assessment Resources and Instructional Support Resources

The resource sites listed below are provided by the GADOE and are designed to support the instructional and assessment needs of teachers. All BLUE links will direct teachers to the site mentioned.

- Georgiastandards.org provides a gateway to a wealth of instructional links and information. Open the ELA/Math tab at the top to access specific math resources for GSE.
- Mathematics Georgia Standards of Excellence (MGSE) Frameworks are "models of instruction" designed to support teachers in the implementation of the GSE. The Georgia Department of Education, Office of Standards, Instruction, and Assessment has provided an example of the Curriculum Map for each grade level and examples of Frameworks aligned with the GSE to illustrate what can be implemented within the grade level. School systems and teachers are free to use these models as is; modify them to better serve classroom needs; or create their own curriculum maps, units and tasks. http://bit.ly/1AJddmx
- The Teacher Resource Link (TRL) is an application that delivers vetted and aligned digital resources to Georgia's teachers. TRL is accessible via the GaDOE "tunnel" in conjunction with SLDS using the single sign-on process. The content is aligned to Georgia Standards of Excellence and National Education Technology Standards and is pushed to teachers based on course schedule.
- The Georgia Online Formative Assessment Resource (GOFAR) accessible through SLDS contains test items related to content areas assessed by the Georgia Milestones Assessment System and NAEP. Teachers and administrators can utilize the GOFAR to develop formative and summative assessments, aligned to the state-adopted content standards, to assist in informing daily instruction.

The Georgia Online Formative Assessment Resource (GOFAR) provides the ability for Districts and Schools to assign benchmark and formative test items/tests to students in order to obtain information about student progress and instructional practice. GOFAR allows educators and their students to have access to a variety of test items - selected response and constructed response - that are aligned to the State-adopted content standards for Georgia's elementary, middle, and high schools.

Students, staff, and classes are prepopulated and maintained through the State Longitudinal Data System (SLDS). Teachers and Administrators may view Exemplars and Rubrics in Item Preview. A scoring code may be distributed at a local level to help score constructed response items.

For GOFAR user guides and overview, please visit: https://www.gadoe.org/Curriculum-Instruction-and-Assessment/Assessment/Pages/Georgia-Online-Formative-Assessment-Resource.aspx

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- Georgia Virtual School content available on our Shared Resources Website is available for anyone to view. Courses are divided into modules and are aligned with the Georgia Standards of Excellence.
- Georgia Milestones Resources are available to provide more information for the Georgia Milestones as provided by the GaDOE.
- Georgia Milestones Assessment System_resources can be found at:
http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Assessment/Pages/Georgia-Milestones-Assessment-System.aspx

Features of the Georgia Milestones Assessment System include:

- Open-ended (constructed-response) items
- Norm-referenced items to complement the criterion-referenced information and to provide a national comparison;
- Transition to online administration over time, with online administration considered the primary mode of administration and paper-pencil as back-up until the transition is complete.


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## Internet Resources

The following list is provided as a sample of available resources and is for informational purposes only. It is your responsibility to investigate them to determine their value and appropriateness for your district. GaDOE does not endorse or recommend the purchase of or use of any particular resource.

## GENERAL RESOURCES

## Illustrative Mathematics

Standards are illustrated with instructional and assessment tasks, lesson plans, and other curriculum resources.

## Mathematics in Movies

Short movie clips related to a variety of math topics.

## Mathematical Fiction

Plays, short stories, comic books and novels dealing with math.

## The Shodor Educational Foundation

This website has extensive notes, lesson plans and applets aligned with the standards.
NEA Portal Arkansas Video Lessons on-line
The NEA portal has short videos aligned to each standard. This resource may be very helpful for students who need review at home.

Learnzillion
This is another good resource for parents and students who need a refresher on topics.

## Math Words

This is a good reference for math terms.

## National Library of Virtual Manipulatives

Java must be enabled for this applet to run. This website has a wealth of virtual manipulatives helpful for use in presentation. The resources are listed by domain.

## Geogebra Download

Free software similar to Geometer's Sketchpad. This program has applications for algebra, geometry, and statistics.

## Utah Resources

Open resource created by the Utah Education Network.

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## RESOURCES FOR PROBLEM-BASED LEARNING

Dan Meyer's Website
Dan Meyer has created many problem-based learning tasks. The tasks have great hooks for the students and are aligned to the standards in this spreadsheet.

## Andrew Stadel

Andrew Stadel has created many problem-based learning tasks using the same format as Dan Meyer.

## Robert Kaplinsky

Robert Kaplinsky has created many tasks that engage students with real life situations.

## Geoff Krall's Emergent Math

Geoff Krall has created a curriculum map structured around problem-based learning tasks.

## ADDITIONAL RESOURCES

Burns, Marilyn. About Mathematics. (1992) Math Solutions Publications. Sausalito, California.
Burns, Marilyn (2007). About Teaching Mathematics: A K-8 Resource. Sausalito, CA: Scholastic, Inc.

Chapin, Suzanne and Johnson, Art (2006). Math matters: Understanding the math you teach $2^{\text {nd }}$ Edition. Sausalito, CA: Math Solutions Publications

Katie Hendrickson, Illuminations Summer Institute., (2012) National Council of Teachers of Mathematics, Reston, Virginia.

National Research Council (2001). Adding it up: Helping children learn mathematics. J. Kilpatrick, J. Swafford, and B. Findell (Eds). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Swetz, Frank and J.S. Hartzler. Mathematical Modeling. (1991) National Council of Teachers of Mathematics. Reston, Virginia.

Van De Walle, John, Elementary and Middle School Mathematics, Teaching Developmentally, (2005).

Van de Walle, John A., Karp, Karen, Bay-Williams, Jennifer (2010). Elementary and middle school mathematics: teaching developmentally $7^{\text {th }}$ edition. Boston: Allyn \& Bacon (Pearson).

